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A TWO-ECHELON INVENTORY/DISTRIBUTION SYSTEM WITH LTL (LESS-THAN-TRUCKLOAD) DELIVERIES – MIXED INTEGER APPROACH

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ABSTRACT: Enhancing the level of management at the company level as well as of its operation at the level of the entire logistical network enforces detailed planning of both the flow of goods between all the network links and the levels of supply of the distributed products. An additional difficulty is the fact that many new ideas pertaining to company operation in a chain, despite requiring decisions at the operational level, are only possible to be evaluated in a long term.In this article two linear programming models are presented, serving for simultaneous planning of transportation and inventory. In both cases LTL (less-than-truckload) deliveries are considered, in which in one route goods are delivered to more than one recipient.

KEY WORDS: supply chain, delivery, less-than-truckload, mixed integer

1 INTRODUCTION

The increase of popularity of management ideas based upon cooperation of companies within a chain of supplies enforces the companies to apply modern methods of planning of simultaneous actions in several areas at a time. At present, an appropriate structure of the transportation system as well as the selection of transportation and inventory policies impacts not only the cost of product delivery but also the level of customer service.

Classical models in which the flows of goods are considered independently from inventory policies are no longer sufficient for the needs of lowering the costs in the entire logistical chain or to guarantee a proper level of service. Models called Joint-Transportation-and-Inventory Problems (JTIP) are used more and more frequently in logistical network operation analysis. In these models, aspects related to the above mentioned areas are considered simultaneously as limitations or decision variables.

The fundamental purpose of this paper is a presentation of a proposal of models serving for determination of the optimal transportation and inventory policies.

2 THE TRANSPORTATION AND INVENTORY PROBLEM

Let I and T signify a vector determining parameters of the applied policy (inventory and transportation, respectively). Furthermore, let $C_t(I,T)$ be the cost of application of a joint policy of inventory I and distribution T in a period t. The class of problems generally referred to as transportation and inventory problems can be formulated in the form of the formulas (1) – (3).

$$\min_{I,T}\left\{\sum_{t=1..H} C_t(I,T)\right\} \qquad (1)$$

$$I \in \Omega \qquad (2)$$

$$T \in \Pi \qquad (3)$$

The target function expressed by the formula (1) provides minimizations – depending on the problem version – of joint or expected costs of joint policies application in the planning horizon H. Thus defined criterion function does not exclude application of other criteria such as minimization of time of delivery or maximization of profit. The limitations (2) and (3) provide that policies (of inventory and transportation, respectively) selected in the solution will be possible to carry out. For a situation with a planning horizon longer than one period, the model requires being extended by limitations ensuring continuity of policies application. For example, the start state of a store in the period i should be in compliance with the end state from the period i-1.

This rather general model formulation allows for numerous detailed problem forms: from simple deterministic models with direct deliveries, through models with LTL deliveries, up to very complex stochastic problems. Due to a high level of complication of thus defined problem, in the approaches appearing frequently in the subject's bibliography the optimization process is carried out in two stages. In the first stage a partial solution is found in one of the analysed areas (inventory or transportation). Afterwards, assuming that the determined solution is the ultimate one, the optimal solution in the second area is found. A solution of an inventory and transportation problem determined in this way is referred to as the optimal inventory policy I with the application of transportation policy T or the optimal transportation policy T with the application of the inventory policy I. The costs of application of such joint policies are noted, respectively, as $C_t(I|T)$ and $C_t(T|I)$.

3 OVERVIEW OF INVENTORY AND TRANSPORTATION MODELS

The necessity of planning actions simultaneously in the areas of inventory and transportation has already been noticed in the early eighties. The first described applications of optimization methods in this area is the system of route planning and scheduling of technical gases supplies described by Bell [1], as well as the use of a simulation model to determine the dependence between decisions pertaining to the areas of inventory and transportation presented by Golden [4]. In the first application the decrease in costs by 6-10% was achieved, while in the second one the decrease amounted to as much as 23% with a simultaneous decrease of shortages by 50% and the increase of the per hour indicator by 8.4%.

The authors of the first comparative research which contained estimations of the scale of savings caused by application of joint models in relation to divisive approach are Federgruen and Zipkin. In their work of 1984 [3] they present a model of vehicle routes determination in which they additionally take into account inventory costs. The results obtained through this formulation guaranteed savings of 6-7% in comparison to the results determined on the basis of the classical model of vehicle routes determination. Analogically, Dror and Ball in the work [2] analyse the problem of fuel oil inventory and delivery. The inventory and transportation policy ensured the improvement in the efficiency of deliveries by over 50% in comparison with the rules in use in the company at that time and over 25% in comparison with other optimization approaches.

4 FORMULATION PROPOSAL

The formulation of inventory and transportation model presented below is a formulation of vehicle routes determination recipients set partition model with a discreet, cyclic supply planning model. The recipients set partition model has been selected for the possibility of precise determination of carried-out routes. Such proceeding is very often put into practice where the decision-maker takes into account in the first place the specificity of the problem and area where transportation is carried out and determines a set of routes possible to carry out.

In the formulation presented by the formulas (4)-(11) the indexes i, j and k are, respectively, recipients, routes and periods. The parameter matrix A contains routes possible to carry out. The element a_{ij} of this matrix has the value of 1, when a recipient i is serviced in a route j and of 0 in the other case. The parameter matrices B and C are, respectively, b_i - the costs of inventory of a goods unit with a recipient i and c_j - the costs of carrying out of a route j. The last parameter matrix is the matrix D containing the recipient demands in a period k. Additionally, the capacity of a vehicle used in transportation is denoted by an L symbol. The parameter M is also used in the formulation, signifying positive value greater than the sum of all recipient demands in the analysed horizon. This parameter is used in the model for technical reasons.

Three groups of decision variables were used in the formulation. The group of binary x_{jk} variables should be enumerated as the first one. They have the value of 1 when a route j is carried out in a period k and of 0 in case of the contrary. Further, two groups of variables related to supplies were used – y_{ik} and z_{ijk}. The first one of them is the magnitude of the supply of a recipient i in a period k, while the second one is the magnitude of a carried out delivery to a recipient i in a route j and in a period k.

$$\min\left[\sum_{j=1..m,k=1..p} c_j x_{j,k} + \sum_{i=1..n,k=1..p} b_i y_{i,k}\right] \tag{4}$$

$$z_{i,j,k} - a_{i,j} x_{j,k} M \le 0 \quad \text{for each} \quad \begin{matrix} i = 1..n \\ j' = 1..m \\ k = 1..p \end{matrix} \tag{5}$$

$$\sum_{i=1..n} z_{i,j,k} \le L \quad \text{for each} \quad \begin{matrix} j = 1..m \\ k = 1..p \end{matrix} \tag{6}$$

$$y_{i,k} + \sum_{j=1..m} z_{i,j,k} - d_i - y_{i,k+1} = 0 \quad \text{for each} \quad \begin{matrix} i = 1..n \\ k = 1..p-1 \end{matrix} \tag{7}$$

$$y_{i,k} + \sum_{j=1..m} z_{i,j,k} - d_i - y_{i,1} = 0 \quad \text{for each} \quad \begin{matrix} i = 1..n \\ k = p \end{matrix} \tag{8}$$

$$x_{j,k} \in \{0,1\} \quad \text{for each} \quad \begin{matrix} j = 1..m \\ k = 1..p \end{matrix} \tag{9}$$

$$y_{i,k} \ge 0 \quad \text{for each} \quad \begin{matrix} i = 1..n \\ k = 1..p \end{matrix} \tag{10}$$

$$z_{i,j,k} \ge 0 \quad \text{for each} \quad \begin{matrix} i = 1..n \\ j = 1..m \\ k = 1..p \end{matrix} \tag{11}$$

The target function expressed by the formula (4) ensures minimizations of joint costs in the entire planning horizon. The first sum component are the transportation costs, while the second one are the inventory costs. The limitation (5) provides the combination of the x_{jk} and z_{ijk} variables. It guarantees that deliveries will be carried out only in periods when a route encompassing a given recipient will be executed. The limitation (6) is responsible for maintaining the maximum vehicle capacity. The formulas (7) and (8) express the limitations guaranteeing the continuity of the inventory policy. Thus, the limitation (7) guarantees continuity in the periods from 1 to p-1. As to the limitation (8), as it is assumed that transportation is cyclic, combines states from the period p with those from the period 1. The limitations (9), (10) and (11) are boundary limitations for the decision variables x, y and z.

The introduction of the decision variable z_{ijk} in the presented model guarantees the optimal delivery magnitudes. However, due to a large number of decision variables in this group (the recipients, routes and periods product) this has negative influence upon the possibilities of solving this formulation. The observation of real transportation systems in which one of the crucial factors in evaluating transportation is the maximum use of vehicle capacity suggests an approach in which not only a recipient's appurtenance to a route will be indicated in a route's definition, but it will also contain the delivery's magnitude. The model expressed by the formulas (12)-(16) instead of the binary parameter a_{ij} uses the e_{ij} parameter, the value of which determines the magnitude of a delivery to a recipient i in a route j. In this formulation two groups of decision variables are used - x_{jk} and y_{ik}. Their interpretation is analogical to the one from the previous formulation.

$$\min\left[\sum_{j=1..m,k=1..p} c_j x_{j,k} + \sum_{i=1..n,k=1..p} b_i y_{i,k}\right] \tag{12}$$

$$y_{i,k} + \sum_{j=1..m} e_{i,j} x_{j,k} - d_i - y_{i,k+1} = 0 \quad \text{for each} \quad \begin{matrix} i = 1..n \\ k = 1..p-1 \end{matrix} \tag{13}$$

$$y_{i,k} + \sum_{j=1..m} e_{i,j} x_{j,k} - d_i - y_{i,1} \geq 0 \quad \text{for each} \quad \begin{matrix} i = 1..n \\ k = p \end{matrix} \tag{14}$$

$$x_{j,k} \in \{0,1\} \quad \text{for each} \quad \begin{matrix} j = 1..m \\ k = 1..p \end{matrix} \tag{15}$$

$$y_{i,k} \geq 0 \quad \text{for each} \quad \begin{matrix} i = 1..n \\ k = 1..p \end{matrix} \tag{16}$$

The limitation (13) ensures the continuity of the inventory policy. The limitation (14) is a modified version of the limitation (8) from the previous model. Its task is to provide cyclicity in the application of the specified inventory policy. In this case, however, due to the magnitudes of deliveries defined as the conditions of the problem which could reduce the set of admissible solutions in a too radical manner, the sign of equality was replaced by the sign of minority. In this situation, the described limitation should make admissible transportation policies in which, in the planning horizon, to each recipient at least deliveries in compliance with the recipient's demand will be carried out.

When proceeding with predefined delivery magnitudes the fundamental problem is the one of the generation of routes. Apart from checking the feasibility of a route and determining its costs before the start of the optimization process it is indispensable to determine the magnitude of deliveries carried out in a route. The use of one of several heuristic rules can be the simplest approach here. An example of such a rule can be the adaptation of magnitude of deliveries which would guarantee the frequency of delivery to all the recipients in a route to be the same. Column generation is a more advanced technique.

The presented formulations are very flexible and constitute a good basis for construction of more complex models. For example, taking into account of different demands for products in subsequent periods or the admission of heterogeneous machine park is not much of a problem. Such changes do not influence the magnitude of the models or the time required to solve them. Even a significant modification as conducting transportation from more than one place involves exclusively increasing the number of considered routes. Nevertheless, it will have impact upon the amount of time necessary for the preparation and the solution of the model. Still, a large number of routes is not a problem which would render the use of this formulation impossible. Analogically with the case of the models involving partitions of the sets of vehicle routes determination recipients, column generation can be used here, greatly increasing the magnitude of solved problems.

5 APPLICATION EXAMPLE

The models presented in the previous part of the paper have been used for solving an exemplary inventory and transportation problem consisting of determination of a weekly transportation plan. The developed plan will be applied in a cyclical manner, hence the assumption that the end state of the store in the last period is equal to its start state in the first period of the analysed horizon. The transportation system consists of 1 distribution centre, from which deliveries are carried out and of 10 recipients with pre-programmed demand. Its magnitude, as well as the coordinates of the recipients are presented in Table 1. The cost of storage of one unit of goods during one period is 1 money unit. As to the costs of covering one distance unit, they amount to 10 money units. The capacity of a vehicle used for transportation is 1000 units. 175 routes, i.e. all routes to 3 recipients or fewer, were taken into account in the optimization process.

Table 1 Demands, locations of the distribution centres and customers.

No.	X	Y	Demand
0	560	370	Distribution centre
1	520	330	200
2	300	400	100
3	370	520	200
4	490	490	300
5	520	640	400
6	200	260	300
7	400	300	200
8	210	470	200
9	170	630	300
10	310	620	100

Source: author's own

The presented problem has been solved with the proposed formulations. The results obtained using the first model are presented in Tables 2 and 3. Table 2 contains scheduled deliveries to all recipients. Table 3 contains store states of each recipient in the analysed horizon. In this case, the joint cost of inventory and transportation amounted to 83 200 money units. For the solved problem this model consisted of 9 675 decision variables, 825 of which were binary variables. The solution of the model was determined in 6 hours using the COIN-OR software.

Table 2 Optimal delivery plan – formulation I.

Recipient	Periods				
	1	2	3	4	5
1	400	0	600	0	0
2	500	0	0	0	0
3	0	0	0	1000	0
4	600	0	900	0	0
5	0	1000	0	1000	0
6	500	0	1000	0	0
7	0	0	1000	0	0
8	0	1000	0	0	0
9	0	0	500	0	1000
10	0	0	500	0	0

Source: author's own

Table 3 Store states in the optimal delivery plan – formulation I .

Recipient	Periods				
	1	2	3	4	5
1	200	0	400	200	0
2	400	300	200	100	0
3	400	200	0	800	600
4	300	0	600	300	0
5	0	600	200	800	400
6	300	0	700	400	100
7	200	0	800	600	400
8	0	800	600	400	200
9	400	100	300	0	700
10	100	0	400	300	200

Source: author's own

Tables 4 and 5 present the results obtained using the second formulation. These tables contain, respectively, delivery plans for all recipients and store states of each recipient in the analysed horizon. In this case, the joint cost of inventory and transportation amounted to 88 500 money units. For the solved problem the model consisted of 875 decision variables, 825 of which were binary variables. Analogically as in the first case, the solution of the model was determined using the COIN-OR software in 2 hours' time.

Table 4 Optimal delivery plan – formulation II .

Recipient	Periods				
	1	2	3	4	5
1	400	0	600	0	0
2	0	0	250	0	250
3	0	0	0	1000	0
4	600	0	900	0	0
5	0	0	1000	0	1000
6	0	0	750	0	750
7	1000	0	0	0	0
8	0	1000	0	0	0
9	0	0	750	0	750
10	0	0	250	0	250

Source: author's own

Due to the specificity of this formulation (predefined delivery magnitudes) the presented solution has been modified in comparison to the one obtained in the optimization process. In order to guarantee the continuity of the inventory policy the deliveries to the recipients 1 and 4 in the period 3 have been diminished from 1 000 pieces by 400 and 100 respectively. As it can be seen in Table 5, this guaranteed that the end state of the store in the period 5 is the same as the start state of the store in the period 1.

Table 5 Store states in the optimal delivery plan – formulation II.

Recipient	Periods				
	1	2	3	4	5
1	200	0	400	200	0
2	100	0	150	50	200
3	400	200	0	800	600
4	300	0	600	300	0
5	400	0	600	200	800
6	300	0	450	150	600
7	800	600	400	200	0
8	0	800	600	400	200
9	300	0	450	150	600
10	100	0	150	50	200

Source: author's own

6 CONCLUSION

The conducted research allowed confirming the possibility of using optimization methods in planning and analysis of inventory and transportation solutions. In the presented example the introduced formulation allowed to determine the optimal delivery strategy taking into account the costs of both the transportation and the inventory.

The fact that the proposed formulations are characterized by great flexibility is their important advantage. Many additional conditions (often resulting from the specificity of a given case) can be easily taken into account in this model.

In case of the proposed models, the size of solved problems is a fundamental problem. An approach widely used in case of a large number of variables in a linear programming problem is the above-mentioned column generation algorithm. As a continuation of the presented research, it is planned to use this approach in solving the introduced formulations. In further research stages it is expected that an approximate algorithm will be developed, guaranteeing solutions of even very large models in an acceptable amount of time, as well as that the rules of delivery plan construction will be determined through analysis of precise results.

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